Deformations of extremal black holes in GR and from stringy interactions

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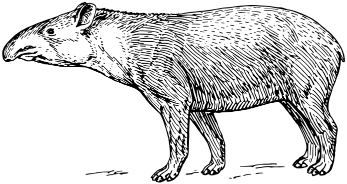

Based on **arXiv:1707.05319** & **arXiv:1802.02159**

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Motivations

GR and beyond-GR

- GR must be corrected at high energies
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Vacuum solution — Black hole

- may hold a key to quantum gravity
- analytical BH solutions are rare in non-GR theories
  e.g. dynamical Chern-Simons BH in slow rotation limit ($a = 0$)

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Deformations of black hole — metric perturbations

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- In weakly-coupled beyond-GR, LEE with source term

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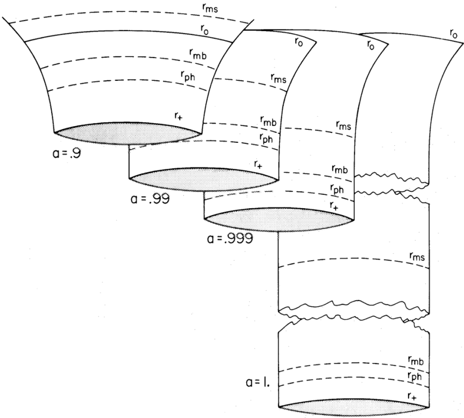
Hard to solve $\Longrightarrow$ need separation of variables

Why NHEK?

Near-horizon extremal Kerr (NHEK)

- a scaling limit of extremal Kerr $(a = M)$
- additional symmetry — $SL(2,\mathbb{R}) \times U(1)$
- separation of variable is possible! [Chen and Stein, PRD 96, 064017]
- can find deformations to NHEK analytically



[Bardeen et al, ApJ 178, 347-370]

Dynamical Chern-Simons and Einstein-dilaton-Gauss-Bonnet

Two example stringy interactions: dCS and EdGB

$$I = \int d^4x \sqrt{-g} \left[\mathscr{L}_{\rm EH} + \mathscr{L}_{\vartheta} + \mathscr{L}_{\rm int} \right]$$

$$\mathscr{L}_{\rm EH} \propto R \qquad \mathscr{L}_{\vartheta} \propto (\partial^a \vartheta)(\partial_a \vartheta)$$

$$\mathscr{L}_{\rm int}^{\rm dCS} \propto \varepsilon \vartheta \, {}^*RR \qquad \mathscr{L}_{\rm int}^{\rm EdGB} \propto \varepsilon \vartheta \, {}^*R^*R$$

- dCS arises from gravitational anomaly cancellation in chiral theories
- EdGB can be derived by expanding the low energy string action to two loops to find the dilaton-curvature interaction

Equation of motion

- take the decoupling limit $\Longrightarrow$ expand everything in powers of ε
- scalar equation of motion

$$\Box\vartheta \propto \varepsilon \begin{cases} {}^{*}RR\,, & \text{dCS} \\ {}^{*}R^{*}R\,, & \text{EdGB} \end{cases}$$

- metric equation of motion

$$G^{(1)}[h_{ab}] = S_{ab}[\vartheta, \vartheta]$$

- solve for the scalar $\vartheta \Longrightarrow$ construct the source term $S_{ab}[\vartheta, \vartheta] \Longrightarrow$ solve for the metric perturbations h_{ab}

Attractor gauge

The "rotating attractor" that makes $SL(2,\mathbb{R}) \times U(1)$ symmetry manifest

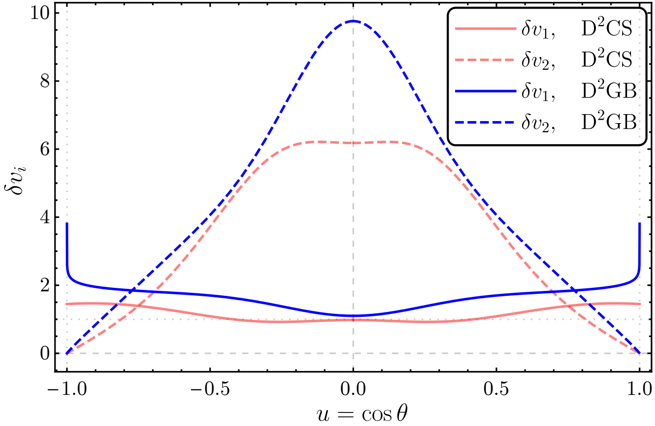
$$\mathrm{d}s^2 = v_1(\theta)\left(-r^2\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{r^2} + \beta^2\mathrm{d}\theta^2\right) + \beta^2 v_2(\theta)(\mathrm{d}\phi - \alpha r\,\mathrm{d}t)^2$$

The leading order metric perturbation is at order ε^2

$$\begin{aligned} v_1(\theta) &\to v_1^{\mathrm{NHEK}}(\theta) + \varepsilon^2\delta v_1(\theta)\,, \qquad & \alpha &\to \alpha^{\mathrm{NHEK}} + \varepsilon^2\delta\alpha\,, \\ v_2(\theta) &\to v_2^{\mathrm{NHEK}}(\theta) + \varepsilon^2\delta v_2(\theta)\,, \qquad & \beta &\to \beta^{\mathrm{NHEK}} + \varepsilon^2\delta\beta\,. \end{aligned}$$

Attractor gauge $\Longrightarrow$ deformed solutions have $SL(2,\mathbb{R}) \times U(1)$ symmetry by construction

Deformed solutions from stringy interactions



For EdGB, there is a true curvature singularity at the poles $u = \pm 1$.
This was first found in Kleihaus et al, PRD 93,044047 numerically.

Properties of solutions

- angular frequency of circular equatorial orbits $r = r_0$
  $\Longrightarrow$ not the frequency measured in asymptotic region
  $\Longrightarrow$ may be constrained observationally for subextremal BH

$$\omega_\phi^{\mathrm{dCS}} = \left[-\frac{3}{4} + \frac{25}{128}\varepsilon^2 + \mathcal{O}\left(\varepsilon^3\right)\right] r_0 \quad \omega_\phi^{\mathrm{EdGB}} = \left(-\frac{3}{4} + \mathcal{O}\left(\varepsilon^3\right)\right) r_0$$

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- coordinate location of deformed horizon $\Longrightarrow$ not deformed
- area of deformed horizon

$$A_{\text{deformed}} = A_{\text{NHEK}} \times \left[1 + \eta\varepsilon^2 + \mathcal{O}(\varepsilon^3)\right]$$

$$\eta_{\text{dCS}} \approx -0.18 \qquad \eta_{\text{EdGB}} \approx +0.26$$

Properties of solutions

- Wald entropy formula [Jacobson et al, PRD 49, 6587]

$$S = -2\pi \oint_{\mathcal{H}} \frac{\delta \mathscr{L}}{\delta R_{abcd}} \hat{\epsilon}_{ab} \hat{\epsilon}_{cd} \bar{\epsilon}$$

entropy $\neq$ horizon area

- macroscopic entropies of extremal BHs in dCS and EdGB, in the decoupling limit

$$S_{\text{deformed}} = S_{\text{NHEK}} \times \left[1 + \xi \varepsilon^2 + \mathcal{O}(\varepsilon^3)\right]$$

$$\xi_{\text{dCS}} \approx +0.49 \qquad \xi_{\text{EdGB}} \approx +1.54$$

positive entropy corrections $\Longleftrightarrow$ extra string degrees of freedom

Takeaway

- metric equations separate in NHEK due to additional symmetries
- analytical deformations to NHEK in many beyond-GR theories can be found in the weakly coupled limit
- macroscopic entropy of the extremal BH solutions can be computed in beyond-GR theories

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Future work

- interpretation of the curvature singularity found in EdGB? a break down of EFT?
- non-extremal BH? $(0 < a < M) \Longrightarrow$ observationally interesting
- microscopic interpretation of the entropies? dual theory? analog of Kerr/CFT?