Gedanken experiment to destroy a BTZ black hole

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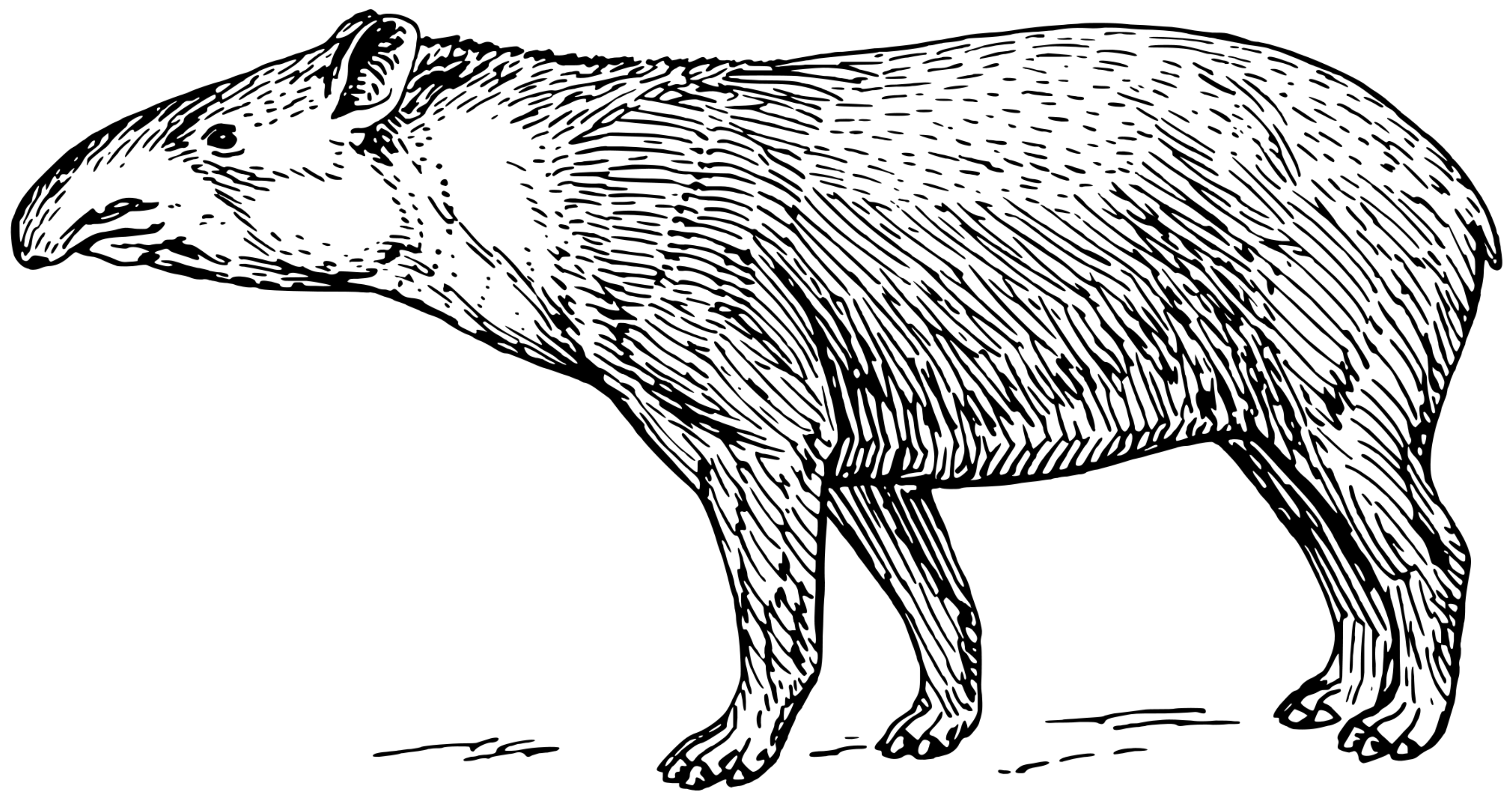

Based on **arXiv:1902.00949** with **Feng-Li Lin** and **Bo Ning**

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Weak cosmic censorship conjecture (WCCC)

- "Nature abhors a naked singularity" (Roger Penrose 1969)
- A general proof is notoriously difficult
- Gedanken experiments in the Kerr-Newman spacetime

Black hole	$M^2 \geq (J/M)^2 + Q^2$
Extremal black hole	$M^2 = (J/M)^2 + Q^2$
Naked singularity	$M^2 < (J/M)^2 + Q^2$

- A way to probe its validity— overspin or overcharge a Kerr-Newman black hole by throwing particles into it

Weak cosmic censorship conjecture (WCCC)

In (3+1)-dimension, provided the null energy condition for the falling matter,

- An extremal Kerr-Newman black hole cannot be overcharged or overspinned (Wald 1974)
- A naked singularity may be created by carefully throwing particles into a near-extremal black hole (Hubeny 1999) self-force? finite-size effect?
- A near-extremal Kerr-Newman black hole cannot be overcharged or overspinned (Sorce & Wald 2017)

gives a conclusive answer to whether WCCC is violated
no self-force calculations

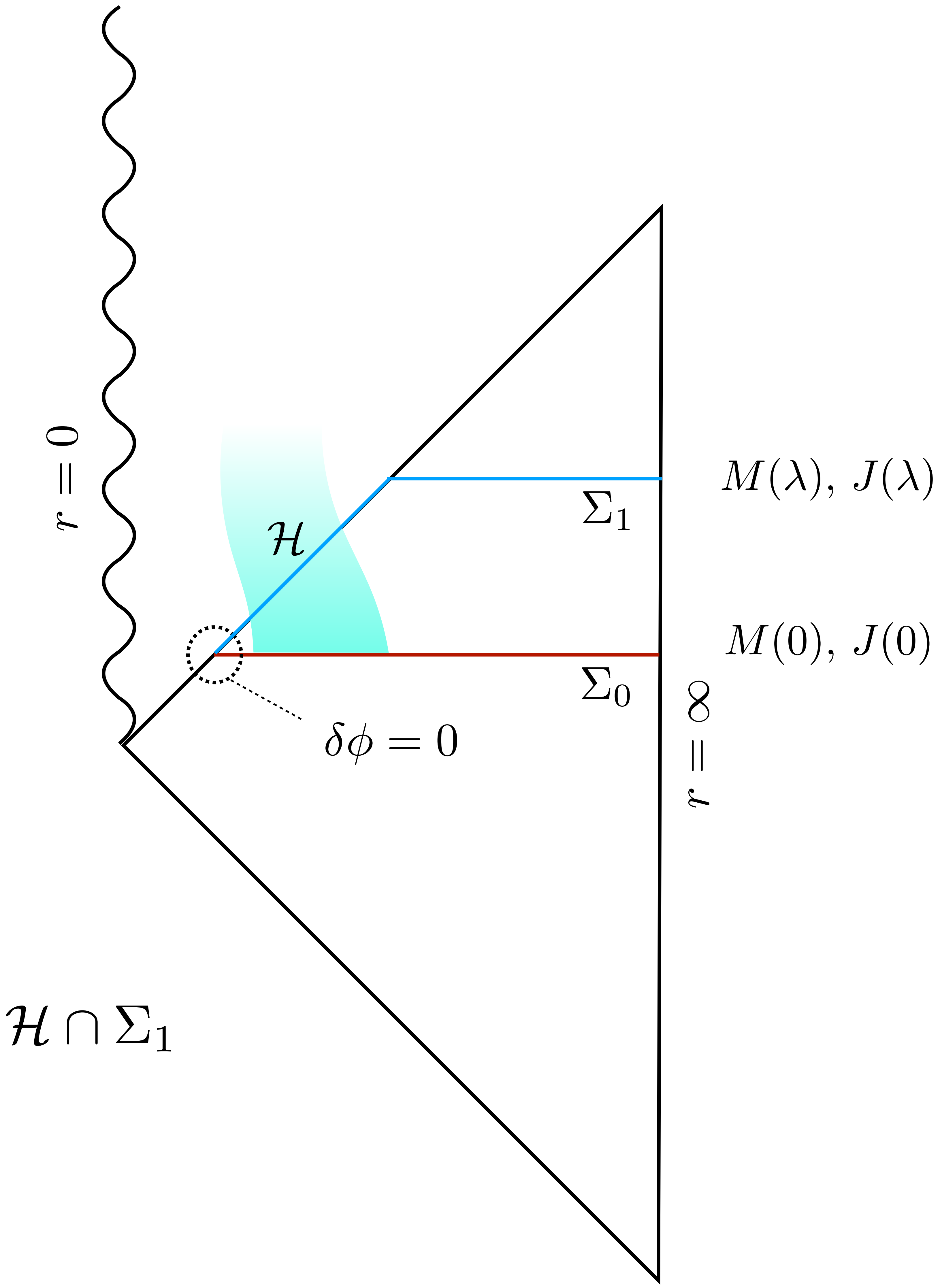
What about gravity in n-dimensions? AdS black holes?

A (2+1)-D AdS black hole —Banados-Teitelboim-Zanelli (BTZ) black hole

- No curvature singularity but a conical singularity
- Described by its mass and angular momentum
- Asymptotically AdS_3 $\longrightarrow$ dual CFT description
- Solutions to a general category of gravity theories in (2+1)-D

Einstein gravity, Chiral gravity (with/without torsion), etc

Gedanken experiment to destroy an extremal BTZ black hole



$\Sigma = \mathcal{H} \cap \Sigma_1$

Linear variational identity

$$\delta M - \Omega_{\rm H}\delta J - T_{\rm H}\delta S = -\int_\Sigma \delta C_\xi$$

"first law" "null energy condition"

We use this identity to constrain the sign of

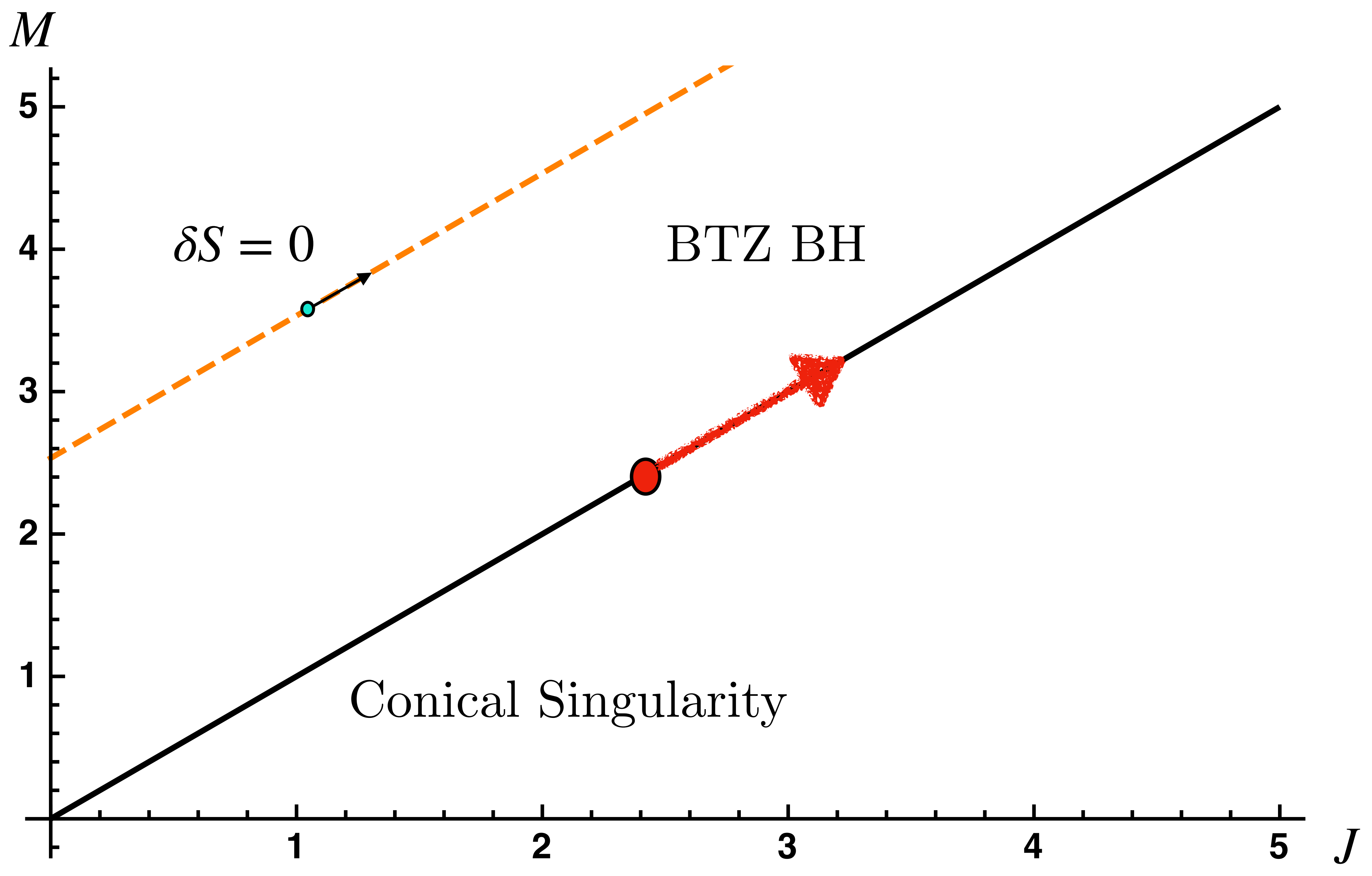
$$f(\lambda) = M(\lambda)^2 + \Lambda_{\rm eff} J(\lambda)^2$$
$$= 2\lambda\sqrt{-\Lambda_{\rm eff}}|J|\left(\delta M - \sqrt{-\Lambda_{\rm eff}}\delta J\right) + \mathcal{O}(\lambda^2)$$

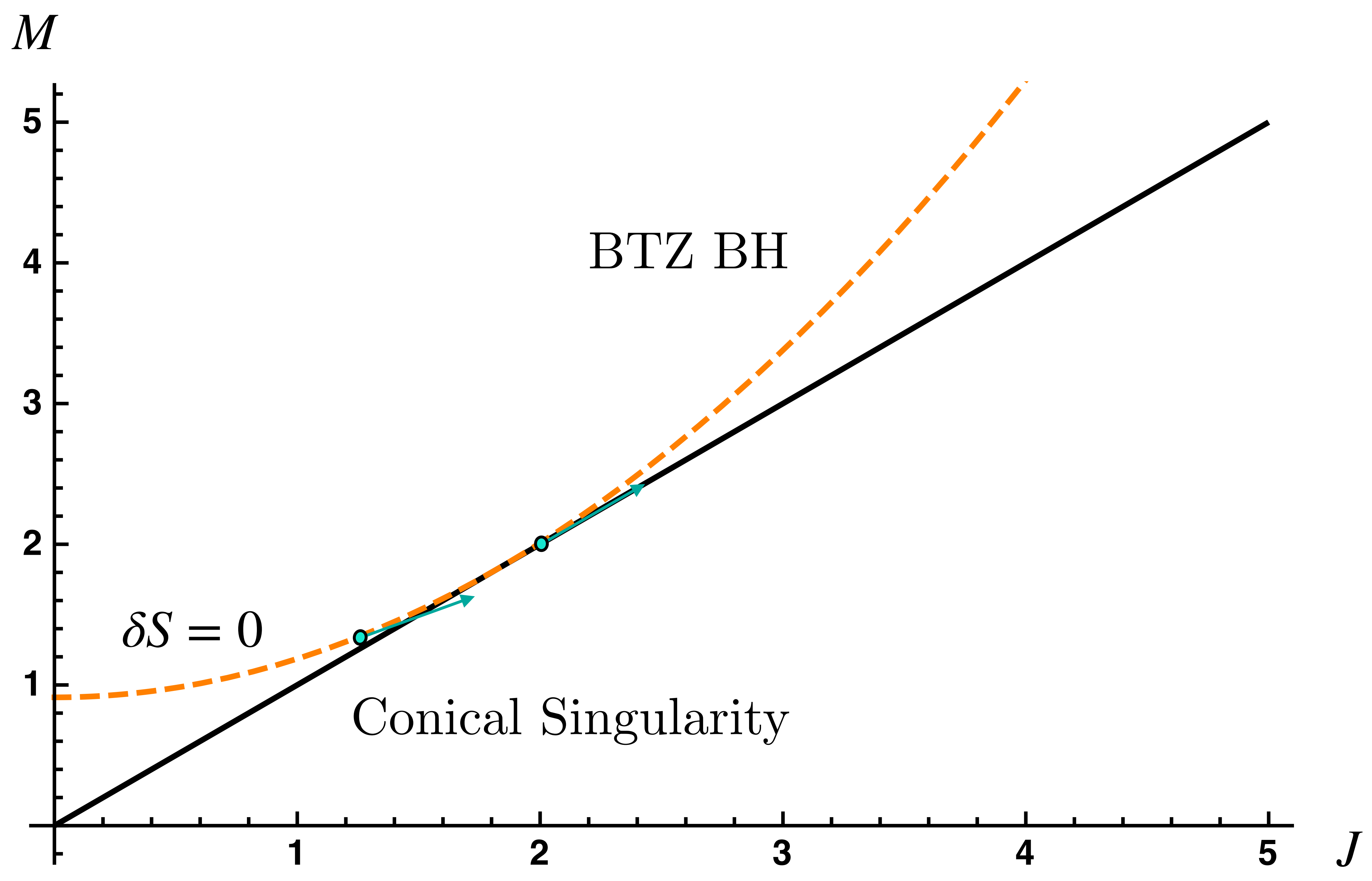
Gedanken experiment to destroy an extremal BTZ black hole

- In **torsional chiral gravity**, whether WCCC holds depends on a relation between the spin angular momentum and its coupling to torsion
- In **chiral gravity**, provided the null energy condition, extremal BTZ cannot be destroyed, thus WCCC is preserved
- In **Einstein gravity**, provided the null energy condition, extremal BTZ cannot be destroyed, thus WCCC is preserved

What about throwing matter into a near-extremal BTZ black hole?

Hubeny-type violations of WCCC





No Hubeny-type violation in chiral gravity

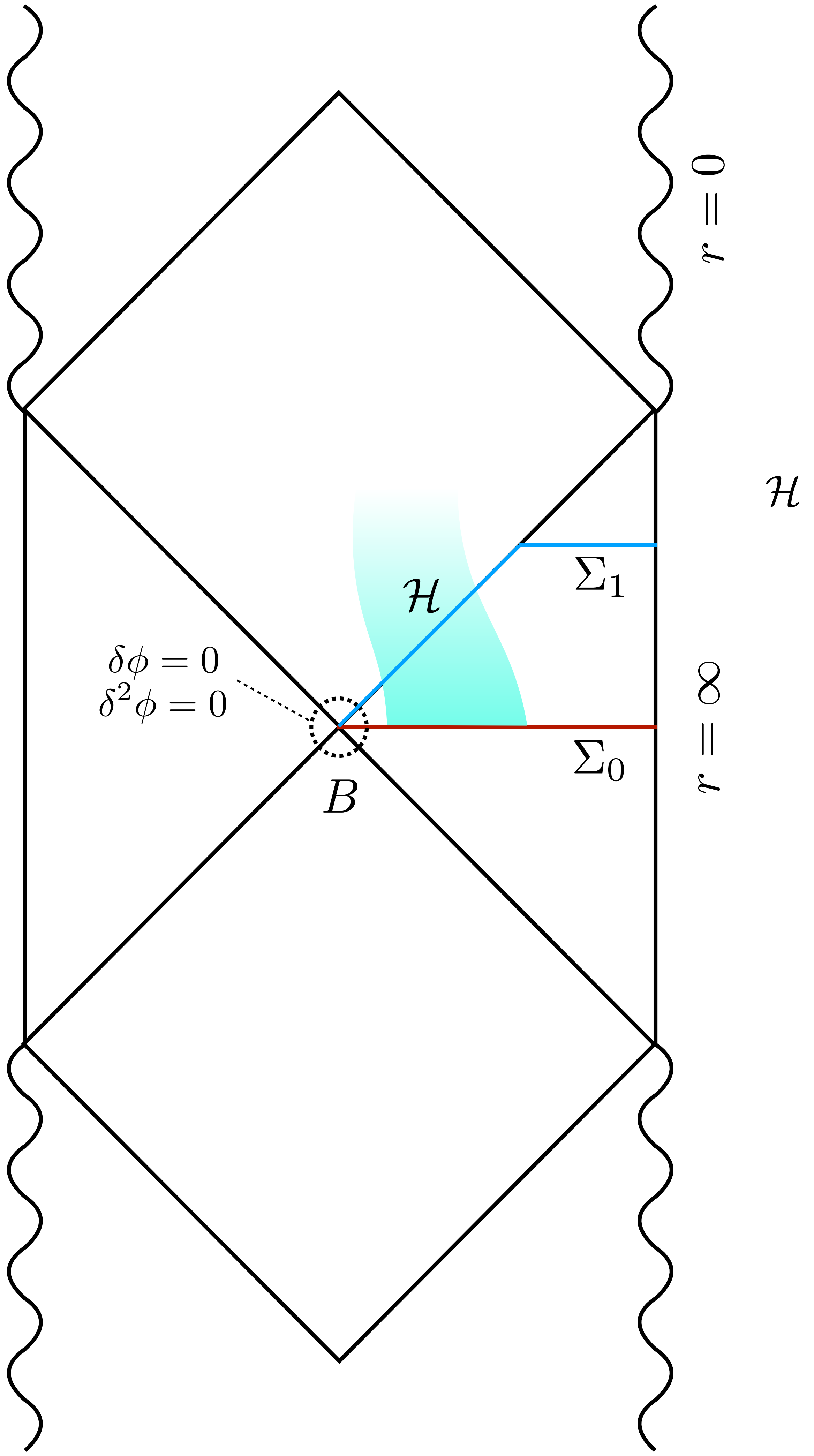
no need to check second order!

Einstein gravity has Hubeny-type violations

need second order variations!

Gedenken experiment to destroy a near-extremal BTZ black hole

$\Sigma = \mathcal{H} \cap \Sigma_1$



Second order variational identity

$$\delta^2 M - \Omega_{\mathrm{H}}\delta^2 J = \mathcal{E}_\Sigma - \int_\Sigma i_\xi(\delta E \wedge \delta\phi) - \int_\Sigma \delta^2 C_\xi$$

"first law" Canonical energy "null energy condition"

We use this identity to constrain the sign of

$$\begin{aligned} f(\lambda) &= M(\lambda)^2 + \Lambda_{\mathrm{eff}} J(\lambda)^2 \\ &= (M^2 + \Lambda J^2) + \lambda g_1(\delta M, \delta J) \\ &\quad + \lambda^2 g_2(\delta M^2, \delta J^2, \delta^2 M, \delta^2 J) + \mathcal{O}(\lambda^3) \\ &= \left(M\varepsilon + \lambda \frac{\Lambda J \delta J}{M}\right)^2 \geq 0 \end{aligned}$$

WCCC is preserved!

The takeaway

- In (2+1)-dimension, provided the null energy condition and torsionless limit, WCCC is preserved for a BTZ black hole with a conical singularity
- WCCC may be violated in presence of torsion
- Our gedanken experiment around BTZ is holographically mapped to the cooling of the boundary CFT
- Our results indicate the third law of thermodynamics holds for the boundary CFT
- Generalizations to higher dimensional AdS black holes can be done in the future

Additional slides

Wald's Lagrangian approach

The Noether current associated with a Lagrangian L and a Killing vector field ξ is

$$j_\xi = \Theta(\phi, \mathcal{L}_\xi \phi) - i_\xi L$$

surface term in variation

According to the Noether theorem, the Noether current can also be written as

$$j_\xi = dQ_\xi + C_\xi$$

"Noether charge" "Constraint of the theory"

Variation of both equations gives the **fundamental linear variational identity**

$$\int_{\partial\Sigma} \delta Q_\xi - i_\xi \Theta(\phi, \mathcal{L}_\xi \phi) = \int_\Sigma \Omega(\phi, \delta\phi, \mathcal{L}_\xi \phi) - \int_\Sigma \delta C_\xi - \int_\Sigma i_\xi (E \wedge \delta\phi)$$

"Simplectic current" equation of motion

Mielke-Baekler model

$$L = L(e^a, \omega^a) = L_{\mathrm{EC}} + L_\Lambda + L_{\mathrm{CS}} + L_{\mathrm{T}} + L_{\mathrm{m}} \longleftrightarrow$$ Motivated by writing the gravity theory as a Poincaré gauge theory

$$L_{\mathrm{EC}} = \frac{1}{\pi} e^a \wedge R_a \longleftrightarrow$$ Einstein-Cartan term

$$L_\Lambda = -\frac{\Lambda}{6\pi}\epsilon_{abc} e^a \wedge e^b \wedge e^c \longleftrightarrow$$ Cosmological term with $\Lambda < 0$

$$L_{\mathrm{CS}} = -\boxed{\theta_L}\left(\omega^a \wedge \mathrm{d}\omega_a + \frac{1}{3}\epsilon_{abc}\omega^a \wedge \omega^b \wedge \omega^c\right) \longleftrightarrow$$ Chern-Simons term

coupling constant

$$L_T = \frac{\boxed{\theta_T}}{2\pi} e^a \wedge T_a \longleftrightarrow$$ Torsion term

Three limits of MB model

(i) Einstein gravity $\theta_{\mathrm{L}} \to 0$ $\theta_{\mathrm{T}} \to 0$

coefficient of the torsion tensor

(ii) Chiral gravity (1) $\mathcal{T}(\theta_{\mathrm{L}}, \theta_{\mathrm{T}}) = \dfrac{-\theta_{\mathrm{T}} + 2\pi^2\Lambda\theta_{\mathrm{L}}}{2 + 4\theta_{\mathrm{T}}\theta_{\mathrm{L}}} \to 0$ (2) $\theta_{\mathrm{L}} \to -1/(2\pi\sqrt{-\Lambda})$

"torsionless limit" "chiral limit"

(iii) Torsional chiral gravity (1) $\theta_{\mathrm{L}} \to -1/(2\pi\sqrt{-\Lambda})$ (2) $\mathcal{T}(\theta_{\mathrm{L}}, \theta_{\mathrm{T}}) = \pi\sqrt{-\Lambda}/2 \neq 0$